Mean reversion in real exchange rate: empirical evidence from Turkey, 1980-1999

Zeynel Abidin Özdemir
Çukurova University, Faculty of Economics and Administrative Sciences, Department of Econometrics, P. O. Box 393, Balcali, Adana 01330 Turkey

Abstract
Purchasing Power Parity (PPP) is one of the most important theoretical relations in international economics. Its empirical measurement, nevertheless, is under discussion. This paper investigates the mean-reversion in bilateral real exchange rate series for lira-dollar (TL/USD), lira-mark (TL/DM), lira-sterling (TL/BP) and lira-franc (TL/FF). The results of the semiparametric estimates show that bilateral real exchange rates have a long memory and are mean reverting processes. On the other hand, the results of the parametric estimates suggest that real TL/USD rate, real TL/DM rate, real TL/BP rate and the real TL/FF rate series are mean reverting processes. Empirical results of both the semiparametric and parametric estimates indicate that the PPP for Turkey can be viewed as a long-run proposition.

1. Introduction
Based on the purchasing power parity hypothesis, national price levels indicated in a common currency should be equal. What this strict hypothesis implies is that movements in the nominal exchange rate should be proportional to the ratio of national price levels and also that the real exchange be constant. The implication of this hypothesis, namely, relative PPP, is that the growth rate in the nominal exchange rate equals the differential between the growth rates in the home and foreign price indices. PPP has been variously named a theory of exchange rate determination, a short-run or long-run equilibrium condition, as well as
an efficient arbitrage condition in the goods or asset markets (Dornbusch, 1988; Taylor, 1995; Taylor and Sarno, 1998). The power parity condition depends on the assumption of perfect inter-country commodity arbitrage. In addition, it is a basic building block of many theoretical and empirical models of exchange rate determination. One may think that PPP is valid only in the long run because of factors like transaction costs, subsidies, taxation, actual or threatened trade restrictions, the existence of nontraded goods, imperfect competition, foreign exchange market interventions, and the differential composition of market baskets and price indices across countries (Obstfeld and Rogoff, 2000).

There has been a vast amount of research concerning the PPP. To start with, researchers like Friedman and Schwartz (1963) and Gaillot (1970) have found that a fairly stable real exchange rate has existed over long periods of time. In the early 1970s, the stronger proposition of continuous PPP was believed to hold. However, this proposition was disputed in the mid to late 1970s (Frankel, 1981). During the 1980s, Adler and Lehman (1983) could not reject the hypothesis of random walk behaviors in real exchange rates in the managed float regime. Taylor (1988), Corbae and Ouliaris (1988), Enders (1988) and Mark (1990) failed to find cointegration between nominal exchange rates and relative prices. Consequent research reduced professional confidence in PPP and led to the common tendency that it had no use empirically (e.g., Dornbusch, 1988). However, for the interwar period, Taylor and McMahon (1988), reported results that support long run purchasing power parity. And for another certain historical period such as the International Gold Standard, Diebold et al. (1991) found results supporting long run purchasing power parity.

A possible cause of the common failure to reject non-stationarity of real exchange rates is that the span of available data for the recent floating rate period alone may simply be too short to provide any proper degree of test power in conventional unit root tests such as ADF (Frankel, 1989; Lothian and Taylor, 1997; Hakkio and Rush, 1991). Parallel to this, Frankel (1989), Abuaf and Jorion (1990), Lothian (1990), Hakkio and Joines (1990) and Lothian and Taylor (1996) have tried to solve this by increasing the sample period. On the other hand, researchers such as Frankel and Rose (1996) have claimed that it may be unreasonable to generate a reasonable level of statistical power with unit root tests, which requires long samples, for many currencies. Due to regime changes, this may also be inappropriate, whereas some authors, Lothian and Taylor (1996), have argued that reliable inferences may be obtained by extending the data across exchange rate regimes. Still, some other authors are skeptical about this view. Mussa (1986) and Frankel (1989) have argued that the statistical properties of the real exchange rate may vary
strongly across nominal exchange rate regimes. During the post-Bretton Woods period, the real exchange rate may have behaved mean-revertingly. Therefore, to settle the issue, the probability of this occurring would require inference based on data for the recent float alone.

Including the use of panel data on exchange rates over shorter periods of time were taken into consideration by some researchers. Flood and Taylor (1996) studied a panel of annual data on 21 industrialized countries over the floating rate period. They have found evidence for the mean reversion towards the long run PPP. Frankel and Rose (1996) have also analyzed a very large panel of annual data on 150 countries in the Post World War II period and obtained similar results. Pedroni (1995), Frankel and Rose (1996), Lothian (1997), Oh (1996), Wu (1996) and Papel and Theodoridis (1998a) have found strong evidence of mean reversion in real exchange rates by implementing panel data variants of conventional unit root tests. However, O'Connell (1998a) has opposed these findings because they have failed to control for cross-sectional dependence in the data. Moreover, Engel et al. (1997), Liu and Maddala (1996), Oh (1996), Papell (1997), Taylor and Sarno (1998), Taylor and Sarno (1998) and Wei and Parsley (1995) have also found similar evidence against reversion to PPP based on a panel of real exchange rates.

An alternative explanation bases the persistence of managed-float deviations from parity on the presence of market frictions getting commodity trade. Dumas (1992), Uppal (1993), Sercu et al. (1995) and Coleman (1995) have developed equilibrium models of real exchange rate determination; they have taken into account transaction costs and have shown that adjusting real exchange rates towards PPP is necessarily a nonlinear process. Market frictions in international trade introduced a neutral range. Within this, deviations from PPP were left uncorrected because they were not large enough to cover transaction costs. In addition, deviations from PPP followed a nonlinear stochastic process, which was mean reverting in this dynamic equilibrium framework.

Michael et al. (1997) have applied an exponential smooth transition autoregression (ESTAR) model to two data sets and have found strong support for the nonlinear representation in an initial test of the hypothesis of the analytic work of PPP adjustment process based on market frictions. Obstfeld and Taylor (1997) and O’Connell and Wei (1997) have reported additional evidence of nonlinear price adjustment induced by the presence of transaction costs. However, O’Connell (1998b), using an equilibrium threshold autoregression (TAR) model for post-Bretton Woods real exchange rates within a panel framework, has found little support for the market-frictions explanations of the persistence of PPP deviations. Sarno (2000) has applied an ESTAR model using data for
Turkey and its major trading partners during the period of 1980-1997. His findings support the long-run PPP for Turkey. Erlat (2003) investigated the persistence in Turkish real exchange rates using unit root tests and autoregressive fractionally integrated moving average models. His findings support the validity of the absolute version of the ‘quasi’ purchasing power parity hypothesis for Turkey. Baum et al. (2001) estimate an ESTAR model of deviations from PPP that are obtained using the Johansen cointegration method. They find the evidence of a mean-reverting dynamic process for sizable deviations from PPP, with the equilibrium tendency varying nonlinearly with the magnitude of disequilibrium.

Cheung and Lai (2001) analyze the dynamics of yen-based real exchange rates during the current float. Cheung and Lai (1998), Koedijk et al. (1998) and Papell and Theodoridis (1998b) have come across with comparable difficulty identifying PPP reversion in real yen rates in contrast to dollar rates. It is different for long historical data (Lothian, 1990). Additionally, it may become difficult to find strong evidence of mean reversion among yen-based real exchange rates, although this may be reduced by including a linear trend variable. A time trend seems at variance with the standard version of long run PPP, which has also been rationalized as capturing the Balassa-Samuelson effect of productivity growth. On the other hand, the difficulty in identifying PPP reversion in real yen rates lies in the intriguing and pertinent dynamics responsible for this. These dynamics are: long-memory dynamics confounding unit-root tests and their ability to distinguish between high and low frequency dynamics. In their study, Cheung and Lai (2001) show when the long-memory dynamics are accounted for, statistical tests based on fractional time series models and also strong evidence for mean reversion can be uncovered in real yen rates. Furthermore, the findings of long memory dynamics may be symptomatic of long-swing dynamics. Lothian (1998) points out the potential implications of long currency swings for PPP analysis. The researcher observes that there can be more behind the PPP re-emergence while studying the dollar exchange rates. Lothian (1998) also emphasizes a potential deficiency of standard time series models in long-swing dynamics. The high and low frequency components can be distinguished effectively; however, this requires better empirical modeling of the long-swing behavior of real exchange rates.

The fact that the data problem caused by long-swing dynamics may afflict the yen rates has been observed by Cheung and Lai (2001). This may be handled via a highly flexible time series model. The real exchange rate behavior, on the other hand, will be modeled by a class of generalized univariate processes, which is called fractionally integrated processes (Granger and Joyeux, 1980; Hosking, 1981). Fractional
dynamics, fractal within nonlinear dynamics, are known to be characterized by long term memory and irregular long cycles, which are embodied as a part of the inherent behavior of fractionally integrated processes (Mandelbrot, 1972). Since these processes are flexible enough to describe large swings and mean-reverting dynamics at the same time, they lend themselves to model the real exchange rate. Actually, since fractional models offer better approximation for the low frequency dynamics than standard time series models, they manage to capture subtle mean reversion (Cheung and Lai, 1993). Cheung and Lai (1993) and Diebold et al. (1991) find mean-reverting long-memory dynamics in long historical series of real exchange rates.

In this article, we analyze the behavior of real bilateral exchange rates of Turkey and its four trading partners during 1980-1999 using the fractional integration analysis. The main contribution of this paper to the existing literature is the application of fractional integration that is a substantially more general approach than standard autoregressive integrated moving average (ARIMA) modelling offers, as it allows one to study a richer class of low frequency dynamics in the series. Classical R/S and modified R/S statistical results show that the real exchange rates have long-term memory. The estimation results show that the real exchange rates of all the series investigated in this study are fractionally integrated series. The results of the Whittle estimates indicate that real exchange rates are mean reverting. A plot of the coefficients of the impulse responses, estimated by optimum fractionally integrated autoregressive moving average (ARFIMA) pointed out for each real exchange rate series, indicate that there is no permanent effect on real exchange series. The findings of the semiparametric estimators show that the series are processes that are both covariance non-stationary and have long-memory. Thus, these series are mean revert processes. These findings support the hypothesis that, for the 1980-1999 period, long run PPP is valid for Turkey.

This paper is organized as follows: the next section briefly describes the ARFIMA models and the long term dependence of a time series and its properties. The third section outlines the testing procedures. The fourth section discusses the data series used in the study and reports the empirical results of the tests. The final section of this article is reserved for conclusions.

2. Fractional statistical analysis

Since the work of Mandelbrot (1972) on fractional processes, Granger and Joyeux (1980), Hosking (1981), Sowell (1990) and other researchers have investigated fractionally integrated processes. These processes belong to long memory due to their ability to reveal significant
dependence between distant observations in time. According to McLeod and Hipel (1978), given a discrete covariance stationary time series process, $y_t$, with autocovariance $\gamma$, the process is long memory if

$$\lim_{T \to \infty} \sum_{j=-T}^{T} |\gamma_j| = \infty$$  \hspace{1cm} (1)$$

is infinite. Another way of characterising these processes is in the frequency domain. Assume that $y_t$ is weakly stationary and let $f(\lambda)$ be the spectral density function of $y_t$ at frequency $\lambda \in (-\pi, \pi]$ satisfying

$$\gamma_j = \int_{-\pi}^{\pi} f(\lambda) \cos(j\lambda) d\lambda \hspace{1cm} j = 0, \pm 1, \pm 2, \ldots$$  \hspace{1cm} (2)$$

where $\gamma_j$ are the autocovariance of $y_t$. Let spectral density of $y_t$ satisfy

$$f(\lambda) \sim c_1 \lambda^{-2d} \hspace{1cm} \text{as} \hspace{0.5cm} \lambda \to 0^+ \hspace{1cm} \text{for} \hspace{0.5cm} 0 < c_1 < \infty$$  \hspace{1cm} (3)$$

and autocovariances follow

$$\gamma_j \sim c_2 j^{2d-1} \hspace{1cm} \text{as} \hspace{0.5cm} j \to \infty \hspace{1cm} \text{for} \hspace{0.5cm} |c_2| < \infty$$  \hspace{1cm} (4)$$

where the symbol $\sim$ means that the ratio of the left hand side and right hand side tends to 1, as $j \to \infty$ in (4), and as $\lambda \to 0^+$ in (3). For $d \in (-0.5, 0.5)$, $y_t$ follows a long memory process (Brockwell and Davis, 1991; Robinson, 1995a, 1995b).

A general class of fractional processes ARFIMA($p, d, q$) is described as

$$\phi(L)(1-L)^d y_t = \theta(L)\varepsilon_t$$  \hspace{1cm} (5)$$

where $\varepsilon_t$ is white noise and $\phi(L) = 1 - \sum_{i=1}^{p} \phi_i L^i$, $\theta(L) = 1 + \sum_{i=1}^{q} \theta_i L^i$ are polynomials in the lag operator $L$ with degrees $p, q$ respectively. All roots of $\phi(L)$ and $\theta(L)$ are stable. The long memory parameter $d$ is not restricted to integer values. For any real number $d > -1$, the fractional difference $(1-L)^d$ is expressed as

$$\begin{align*}
(1-L)^d &= \sum_{k=0}^{\infty} (-1)^k \binom{d}{k} L^k = 1 - dL + \frac{d(d-1)}{2} L^2 + \ldots, \\
\end{align*}$$  \hspace{1cm} (6)$$

The fundamental properties of $y_t$ may be stated in terms of the long memory parameter. When $p = q = 0$, $y_t$ becomes a simple fractional noise process. The process $\varepsilon_t$ could be a stationary and invertable ARMA sequence when its autocovariance decays exponentially. But, it could decay much slower than exponentially. When $d = p = q = 0$ in (5), $y_t = \varepsilon_t$. Thus, $y_t$ is ‘weakly autocorrelated’ and also termed ‘weakly dependent’.
The variance of the process is finite when \( d < 0.5 \), but infinite when \( d \geq 0.5 \). The process is stationary for \( d < 0.5 \) and invertable for \( d \leq -0.5 \). If \( d \in (0, 0.5) \), \( y_t \) is covariance stationary, but its lag-\( j \) autocovariance \( \gamma_j \) decreases very slowly, as in the power law \( j^{2d-1} \) as \( j \to \infty \) in (4). For \( d \in (-0.5, 0) \), \( y_t \) is called antipersistent or intermediate memory. When \( d \leq -0.5 \), \( y_t \) is covariance stationary but not invertable. For \( d \geq 0.5 \), \( y_t \) is nonstationary and has infinite variance. A particularly interesting interval for \( d \) within macroeconomic applications is \( 0.5 < d < 1 \), where the time series \( y_t \), a mean-reverting process, has infinite variance and displays strong persistence (Granger and Joyeux, 1980; Hosking, 1981).

The mean-reverting property of \( y_t \) depends on whether \( d < 1 \). The impact of a shock is known to be persistent forever when \( d = 1 \). The effect of any shock on the fractionaly integrated process with \( d < 1 \) slowly dies out. This can be seen by studying the moving average representation for \((1-L)y_t\):

\[
(1-L)y_t = A(L)e_t
\]

(7)

where \( A(L) = 1 + \theta_1 L + \theta_2 L^2 + \ldots \), obtained from \( A(L) = (1-L)^{-d}\Phi(L) \) with \( \Phi(L) = \phi(L)/\theta(L) \). The moving average coefficients \{\theta_1, \theta_2, \theta_3, \ldots\} are called the impulse responses. The impact of a unit innovation at time \( t \) on the value of \( y_t \) at \( t+j \) is equal to \( (C_\infty = 1 + \theta_1 + \theta_2 + \theta_3 + \ldots \theta) \). As \( j \to \infty \), \( C_\infty = A(1) \). That is the measure of the long run impact of the innovation (Campbell and Mankiw, 1987). Cheung and Lai (1993) show that for the fractionally integrated process with \( d < 1 \), \( C_\infty = 0 \) implies no long run impact of the innovation on the value of \( y_t \). For \( d \geq 1 \), \( C_\infty \neq 0 \). This means that the \( y_t \) process is not mean-reverting since an innovation has a permanent effect on the value of \( y_t \). When \( d < 1 \), the \( y_t \) process is mean-reverting.

3. Testing procedures

Before considering some semiparametric estimates, it should be mentioned that the estimates are based on the so-called adjusted rescaled range or \( R/S \) statistic developed by Hurst (1951) and popularized by Mandelbort (1972), and defined as

\[
R/S = \max_{1 \leq j < T} \sum_{t=1}^{j} (y_t - \bar{y}) - \min_{1 \leq j < T} \sum_{t=1}^{j} (y_t - \bar{y}) \left( \frac{1}{T} \sum_{t=1}^{T} (y_t - \bar{y})^2 \right)^{0.5}
\]

(8)

where \( \bar{y} \) is the sample mean of the process \( y_t \). The specific estimate of \( d \) is given by:
\[ d = \frac{\log(R/S)}{\log(T)} - 0.5. \]  

(9)

Its properties were analyzed in Mandelbrot and Wallis (1969), Mandelbrot (1972, 1975) and Mandelbrot and Taqu (1979) (Beran, 1994).

In spite of the fact that \( R/S \) statistic is able to detect long range dependence, further studies may find that it is sensitive to short range dependence. Lo (1991) proposes a modified \( R/S \) statistic in which short run dependence is incorporated into its denomination. This results in a consistent estimate of the variance of the partial sum in Equation (8), defined as

\[ Q_{\alpha q} = \frac{\max \sum_{r=1}^{T} (y_r - \bar{y}) - \min \sum_{r=1}^{T} (y_r - \bar{y})}{\left( \frac{1}{T} \sum_{r=1}^{T} (y_r - \bar{y})^2 + 2T \sum_{r=1}^{T} \left(1 - \frac{r}{T} \right) r \right)^{1/2}}, \quad q < T \]  

(10)

where \( r_i \) is the \( r \)-th sample autocorrelation. The assumptions and technical details are given to allow the asymptotic distribution of \( Q_{\alpha q} \) to be obtained by Lo (1991). Under the null hypothesis that \( y_i \) is not long-range dependent, \( V \sim T^{-0.5} Q_{\alpha q} \) converges in distribution to a well-defined random variable, whose distribution function is given. The \( V \) statistic is consistent against a class of long-range dependent alternatives, including all ARFIMA models with an example \( d \in [-0.5, 0.5] \). \( V \) diverges in probability to infinity when positive strong dependence is present: \( d \in [0, 0.5] \). On the other hand, it converges in probability to zero when negative strong dependency, \( d \in [-0.5, 0] \), exists (Lo, 1991). So, using the modified \( R/S \) statistics that is denoted by \( Q_{\alpha q} \), the estimates of \( d \) is given by:

\[ d = \frac{\log(Q_{\alpha q})}{\log(T)} - 0.5. \]  

(11)

3.1. The Whittle approximate maximum likelihood estimator

The Whittle estimator (Whittle, 1951) is obtained by maximizing the approximation of the likelihood function in the frequency domain. In this method, the parameter vector \( \theta = (\alpha_1, \ldots, \alpha_p, d, \beta_1, \ldots, \beta_q) \) is estimated by minimizing the following approximate log likelihood function

\[ \log L_{\theta \sigma^2} (\theta, \sigma^2) = -\sum_{j=1}^{m} \log f(\lambda_j | \theta, \sigma^2) - \frac{1}{2\pi} \sum_{j=1}^{m} \frac{I(\lambda_j)}{f(\lambda_j | \theta, \sigma^2)} \]  

(12)
where \( I(\lambda_j) \) is the periodogram defined at the \( j \)th Fourier frequency, 
\[ \lambda_j = \frac{2\pi j}{T}, j = 1, \ldots, m \]

\[
I(\lambda_j) = T^{-1} \sum_{t=1}^{T} (y_t - \bar{y})e^{ij\lambda_j}^2 \tag{13}
\]
m = \((T - 1)/2\), where \([\cdot]\) is the integer part. The reduced form of \( L_w \) with respect to the error variance \( \sigma_w^2 \) is

\[
\log L_w(\theta) = m \log(2\pi) - m \log \left[ \frac{1}{m} \sum_{j=1}^{m} \frac{I(\lambda_j)}{g(\lambda_j)} \right] - \sum \log g(\lambda_j) - m \tag{14}
\]

with \( \sigma_w^2 = \sigma_u^2 \)
\[
\sigma_u^2 = m^{-1} \sum_{j=1}^{m} \frac{I(\lambda_j)}{g(\lambda_j)}, \tag{15}
\]

where \( f(\lambda_j) = \sigma_u^2 g(\lambda_j) / (2\pi) \) with \( g(\lambda) = g(\lambda | \theta) \). In this paper, the parameters of each ARFIMA(\( p, d, q \)) models for real exchange rate series are estimated by reduced form of \( L_w \) (Hauser, 1999).

### 3.2. Log periodogram regression

Geweke and Porter-Hudak (1983) suggest that a semiparametric procedure can be obtained by an estimate of the fractional differencing parameter \( d \). This semiparametric estimator is based on the following regression:

\[
Y_j = \alpha - dZ_j + \epsilon_j, \quad j = 1, 2, 3, \ldots, m \tag{16}
\]

where \( Y_j = \log(I(\lambda_j)), Z_j = \log(4\sin^2(0.5\lambda_j)) \) and \( \epsilon_j \sim i.i.d.(0, \pi^2 / 6) \). \( (\pi^2 / 6) \) is the variance of the asymptotic distribution of \( \epsilon_j \). \( d \) can be estimated by least squares regression of \( Y_j \) on \( Z_j, j = 1, 2, \ldots, m \), where \( m \) is a function of \( T \) such that \( (mT^2 / T) \to 0 \) as \( T \to \infty \). In the linear regression (16),

\[
I(\lambda_j) \text{ is defined as } I(\lambda_j) = \frac{1}{2\pi T} \sum_{j=1}^{T} Y_j e^{ij\lambda_j}^2 \text{ where } \lambda_j = (2\pi j / T) \text{ and } j = 1, 2, \ldots, m. \text{ Geweke and Porter-Hudak (1983) argue that when the } d \text{ parameter is within the } (-0.5, 0) \text{ interval, there exists a sequence } m \text{ such that } ((\log T)^2 / mT) \to 0 \text{ as } T \to \infty
\]

\[
\hat{d} \sim AN \left( d, \pi^2 / (6\sum_{j=1}^{m} (Z_j - \bar{Z})^2) \right) \text{ as } T \to \infty \tag{17}
\]
where $\bar{Z} = T^{-1} \sum_{j=1}^{T} Z_j$ (Brockwell and Davis, 1991).

The modified form of the GPH estimate of the long memory parameter, $d$, of a time series proposed by Phillips (1999a, 1999b) addresses the unit root case, which is not the case in the previous studies. Phillips (1999a) shows that the GPH estimator is inconsistent while exhibiting asymptotic bias toward unity. This weakness of the GPH estimator is solved by Phillips’ modified log periodogram regression estimator where the dependent variable is modified to reflect the distribution of $d$ under the null hypothesis that $d = 1$. The estimator proposes a rise to a test statistic, a standard normal variate under the null for $d = 1$. Phillips (1999b) suggests that deterministic trends should be removed from the series before the estimator is applied. Phillips’ modified GPH log periodogram estimator is given by the following formula,

$$\hat{d} = 0.5 \frac{\sum_{j=1}^{m} y_j \log I_u(\lambda_j)}{\sum_{j=1}^{m} y_j^2}$$

(18)

where $y_j = \left\{ \log \left| -e^{i\lambda_j} \right| - m^{-1} \sum_{j=1}^{m} \log \left| -e^{i\lambda_j} \right| \right\}$. Phillips (1999b) shows that the distribution of $\hat{d}$ follows $\sqrt{m}(\hat{d} - d) \rightarrow_d N(0, \pi^2 / 24)$.

3.3. The Robinson Gaussian Semiparametric Estimator

Robinson (1995a) suggests a Gaussian semiparametric estimator (GSP) to estimate the fractional differencing parameter $d$. The GSP estimator involves the introduction of an additional parameter $m$, which can be taken less than or equal to $[(T-1)/2]$ and should provide $(1/m + m/T) \rightarrow 0$ as $T \rightarrow \infty$. The GSP estimator of $d$ is obtained to minimize the function

$$r(d) = q(\hat{g},d) - 1 = \log m^{-1} \sum_{j=1}^{m} \frac{I(\lambda_j)}{\lambda_j^{-2d}} - 2d m^{-1} \sum_{j=1}^{m} \log \lambda_j$$

(19)

where $q(\hat{g},d) = m^{-1} \sum_{j=1}^{m} \left( \frac{I(\lambda_j)}{\hat{g} \lambda_j^{-2d}} + \log \hat{g} \lambda_j^{-2d} \right)$ with $\hat{g} = m^{-1} \sum_{d=1}^{m} \hat{\lambda}_j^{2d} I(\lambda_j)$.

The value $\hat{d}$ which minimizes $r(d)$ converges in probability to the actual value of $d$ as $T \rightarrow \infty$. Robinson (1995a) shows that $m^{0.5}(\hat{d} - d) \rightarrow_d N(0,0.25)$ as $T \rightarrow \infty$. The asymptotic variance of $\hat{d}$ is equal to $(1/4m)$.
For the GPH, GSP and PMGPH estimators, the choice of the bandwidth parameter $m$ is important. Geweke and Porter-Hudak (1983) proposed that the bandwidth parameter $m$ is chosen from the interval $[T^{0.5}, T^{0.6}]$ for the GPH estimator. However, Hurvich et al. (1998) prove that the optimal bandwidth parameter $m$, that which minimizes the mean squared error, is of order $T^{4/5}$. This is the upper rate for its class of estimators. But, this goes with the stationary region $d \in (0, 0.5)$ and most macroeconomic time series are found to be nonstationary. The optimal $m$ is given by

$$m_{opt}^* = \left( \frac{27}{128\pi^2} \right)^{1/5} \left| \frac{f(0)}{f'(0)} \right|^{2/5} T^{4/5}$$

(20)

where $f(\cdot)$ denotes the I(0) component of the spectral density of series. Instead of $f(\cdot)$ with the spectral density of AR(1) process, a data-dependent parameter selection is given by

$$m^* = \left( \frac{27}{128\pi^2} \right)^{1/5} \left| \left(1 - \hat{r}\right)^2 \right|^{-2/5} T^{4/5}$$

(21)

where $\hat{r}$ is the estimated first-order autocorrelation of the series. $m^*$ is truncated as follows:

$$m = \begin{cases} \lfloor m \rfloor, & \text{if } m^* < m \\ \lfloor m^* \rfloor, & \text{if } m \leq m^* \leq \bar{m} \\ \bar{m}, & \text{if } m > m^* \end{cases}$$

(22)

where $\lfloor z \rfloor$ denotes the integer part of $z$, $\bar{m} = 0.06T^{4/5}$ and $\bar{m} = 1.2T^{4/5}$. These boundaries hold with $2 \leq m \leq T/2$ for $T \geq 100$ (Dittmann, 2000; Hurvich and Deo, 1999).

4. Data and empirical estimates

The data examined are monthly observations from 1980:01 to 1999:12, taken from the IMF’s international financial statistics. The data comprises observations on consumer price indices (CPIs) for Turkey, the US, the UK, Germany and France, and the end of period nominal bilateral exchange rates for the Turkish lira, the US dollar, the UK sterling, German mark and French franc. Monthly real exchange rate series for lira-dollar (TL/USD), lira-sterling (TL/BP), lira-mark (LR/DM) and lira-franc (TL/FF) were constructed from these data in logarithmic form according to the identity $q_t = s_t + p^*_t - p_t$, using the triangular arbitrage condition. In the logarithm of the real exchange rate, $q_t$, $s_t$ denotes the
logarithm of the nominal exchange rate observed at time \( t \), and \( p_t^* \) and \( p_t \) are the logarithms of the foreign and domestic price levels, respectively.

Our first concern is to calculate the classical \( R/S \) and modified \( R/S \) statistics. What we later on estimate is \( d \) based on the classical \( R/S \) and modified \( R/S \) statistics, i.e., \( d \) in equation (5) for the original real exchange rate series and their first and second differences. Their results are given in Table 1 and Table 2 below. Upon looking at the undifferenced series, estimated values of \( d \) based on classical \( R/S \) and modified \( R/S \) statistics are within \((-0.334, -0.172)\) interval. As can be seen, the rates of reversion estimated without taking the differences of the series are meaningless. Therefore, the rates of reversion are indicated by estimating, first of all, the first difference of the series, and then estimating the second difference of the series. Looking at the first differences, the estimated values of \( d \) are around \(-0.4\). This indicates that the original real exchange rates may be fractionally integrated with \( d \in (0.5, 0.6) \). Finally, the results for the second differenced series show that \( d \) is ranging between \(-0.548 \) and \(-0.501 \), suggesting estimates for the undifferenced series between 1.452 and 1.499.

### Table 1
Rescaled range tests results of Reel Exchange Rate

<table>
<thead>
<tr>
<th></th>
<th>( \tilde{V}_n )</th>
<th>( \tilde{V}_n(q) )</th>
<th>Bias</th>
</tr>
</thead>
<tbody>
<tr>
<td>TL/USD</td>
<td>4.751*</td>
<td>2.482*</td>
<td>91.418</td>
</tr>
<tr>
<td>TL/DM</td>
<td>6.035*</td>
<td>3.116*</td>
<td>93.677</td>
</tr>
<tr>
<td>TL/BP</td>
<td>4.337*</td>
<td>2.263*</td>
<td>91.648</td>
</tr>
<tr>
<td>TL/FF</td>
<td>5.178*</td>
<td>2.676*</td>
<td>93.497</td>
</tr>
</tbody>
</table>

\( \tilde{V}_n \) and \( \tilde{V}_n(q) \) denote the classical and modified rescaled statistics, respectively. * indicates significance at the 5% level. The critical value is 1.747 (see Lo, 1991: 1288 for details). Lo (1991) recommends choosing \( q \) as \([T^{0.25}]\). The % bias is computed using the formula \( |(\tilde{V}_n / \tilde{V}_n(q)) - 1| \times 100\), indicating the bias of the classical rescaled range statistics in the presence of short-term dependence.
Table 2
Estimates of Real Exchange Rate $d$ based on R/S statistics

<table>
<thead>
<tr>
<th></th>
<th>$\hat{V}_n$</th>
<th>$\hat{V}_n(q)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Level</td>
<td>First Differences</td>
</tr>
<tr>
<td>TL/USD</td>
<td>-0.215</td>
<td>-0.471</td>
</tr>
<tr>
<td>TL/DM</td>
<td>-0.172</td>
<td>-0.477</td>
</tr>
<tr>
<td>TL/BP</td>
<td>-0.232</td>
<td>-0.464</td>
</tr>
<tr>
<td>TL/FF</td>
<td>-0.199</td>
<td>-0.438</td>
</tr>
</tbody>
</table>

For the parametric model estimates, we present results for the ARFIMA ($p, d, q$) model with $p, q \leq 3$ for each of the real Turkish lira exchange rates. Thus, sixteen parametric model estimates are estimated. Estimation of ARFIMA($p, d, q$) models with $p, q \geq 4$ are also tried. However, the estimations of these models have a high Schwarz information criterion (SIC) value; therefore, the results for these higher order ARFIMA($p, d, q$) models with $p, q \geq 4$ are not reported. In brief, only the results of the ARFIMA($p, d, q$) model that has the minimum SIC is presented. The parameters of ARFIMA($p, d, q$) models are estimated using the Whittle approximate maximum likelihood method. The SIC values of estimated ARFIMA($p, d, q$) models are given in Table 3. The $d$ parameter estimates obtained from the estimated ARFIMA models for each of the real exchange rate series of Turkey investigated in this study are reported in Table 4. Table 4 shows that the estimated $d$ values of other real exchange rate series are around 0.86. Therefore, these real exchange rate series are covariance nonstationary, but are mean revert processes.

As mentioned in Section 3, in estimating $d$ for each of the Turkish real exchange rate series with the GPH, GSP and PMGPH semiparametric methods, the choice of the bandwidth parameter $m$ is determined by the plug-in selection method. These choices vary with the sample size $T$ and the estimated first-order autocorrelation of the real exchange rate series. The GPH, GSP and PMGPH semiparametric estimation results of $d$ of real exchange rate series are reported in Table 5. Table 5 shows that under the $m$ values, the estimated values of $d$ for the real exchange rate series studied in this article are within (0.8, 1) interval. This shows us that the real exchange rate series are covariance
Table 3
Model Selection Criteria of ARFIMA($p, d, q$) Models for Real Exchange Rates

<table>
<thead>
<tr>
<th>SIC for ARFIMA($p, d, q$) Models</th>
<th>TL/USD</th>
<th>TL/DM</th>
<th>TL/BP</th>
<th>TL/FF</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARFIMA($p, d, q$) Models</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0, d, 0)</td>
<td>-2309.91</td>
<td>-2308.49</td>
<td>-2306.86</td>
<td>2309.24</td>
</tr>
<tr>
<td>(0, d, 1)</td>
<td>-2311.41</td>
<td>-2310.45</td>
<td>-2307.84</td>
<td>-2309.95</td>
</tr>
<tr>
<td>(0, d, 2)</td>
<td>-2310.85</td>
<td>-2309.63</td>
<td>-2308.36</td>
<td>-2308.75</td>
</tr>
<tr>
<td>(0, d, 3)</td>
<td>-2308.93</td>
<td>-2307.91</td>
<td>-2306.89</td>
<td>-2307.24</td>
</tr>
<tr>
<td>(1, d, 0)</td>
<td>-2312.21</td>
<td>-2311.14</td>
<td>-2308.93</td>
<td>-2310.34</td>
</tr>
<tr>
<td>(1, d, 1)</td>
<td>-2310.11</td>
<td>-2309.01</td>
<td>-2307.02</td>
<td>-2308.21</td>
</tr>
<tr>
<td>(1, d, 2)</td>
<td>-2310.81</td>
<td>-2307.66</td>
<td>-2306.42</td>
<td>-2306.86</td>
</tr>
<tr>
<td>(1, d, 3)</td>
<td>-2309.40</td>
<td>-2307.87</td>
<td>-2307.11</td>
<td>-2308.03</td>
</tr>
<tr>
<td>(2, d, 0)</td>
<td>-2310.13</td>
<td>-2309.01</td>
<td>-2307.17</td>
<td>-2308.22</td>
</tr>
<tr>
<td>(2, d, 1)</td>
<td>-2308.47</td>
<td>-2307.22</td>
<td>-2305.97</td>
<td>-2306.45</td>
</tr>
<tr>
<td>(2, d, 2)</td>
<td>-2306.79</td>
<td>-2305.82</td>
<td>-2304.31</td>
<td>-2305.00</td>
</tr>
<tr>
<td>(2, d, 3)</td>
<td>-2307.31</td>
<td>-2305.79</td>
<td>-2302.18</td>
<td>-2302.05</td>
</tr>
<tr>
<td>(3, d, 0)</td>
<td>-2309.18</td>
<td>-2308.04</td>
<td>-2306.66</td>
<td>-2307.54</td>
</tr>
<tr>
<td>(3, d, 1)</td>
<td>-2309.38</td>
<td>-2307.81</td>
<td>-2307.14</td>
<td>-2307.98</td>
</tr>
<tr>
<td>(3, d, 2)</td>
<td>-2307.32</td>
<td>-2305.19</td>
<td>-2305.14</td>
<td>-2305.91</td>
</tr>
<tr>
<td>(3, d, 3)</td>
<td>-2302.82</td>
<td>-2301.67</td>
<td>-2301.43</td>
<td>-2309.07</td>
</tr>
</tbody>
</table>

Notes: The table reports the estimates of the ARFIMA($p, d, q$) models selected by the Schwarz Information Criterion (SIC). SIC is equal to $-2 \ln L + (\log n)(p+q+2)$, for $p, q \leq 3$, where $L$ is the Whittle likelihood function, as given in Hauser (1999). The estimates of the mean and residual variance are added, as well as the penalty of two in addition to $p+q$, in the SIC. All ARFIMA models are estimated using the reduced form of the Whittle frequency domain approximate maximum likelihood method.

Table 4
Parameter Estimates of Best ARFIMA($p, d, q$) Models for Real Exchange Rates

<table>
<thead>
<tr>
<th>Series</th>
<th>log-lik</th>
<th>$d$</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>SIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>TL/USD</td>
<td>1160.37</td>
<td>0.870</td>
<td>0.236</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-2312.21</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.094)</td>
<td>(0.117)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TL/DM</td>
<td>1159.84</td>
<td>0.868</td>
<td>0.243</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-2311.14</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.095)</td>
<td>(0.118)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TL/BP</td>
<td>1158.73</td>
<td>0.868</td>
<td>0.237</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-2308.93</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.095)</td>
<td>(0.117)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TL/FF</td>
<td>1159.84</td>
<td>0.869</td>
<td>0.243</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-2311.14</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.050)</td>
<td>(0.118)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: The estimates are for the models that have the minimum SIC. Standard errors of the estimated parameters, in parentheses, are calculated under the asymptotic formula in Robinson (1994) and Beran (1995).
Table 5
Long Memory Parameter $d$ Estimation Results of Reel Exchange Rates

<table>
<thead>
<tr>
<th>Test</th>
<th>TL/USD $(m=[T^{0.735}])$</th>
<th>TL/DM $(m=[T^{0.738}])$</th>
<th>TL/BP $(m=[T^{0.740}])$</th>
<th>TL/FF $(m=[T^{0.735}])$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPH</td>
<td>0.851*</td>
<td>0.951*</td>
<td>0.873*</td>
<td>0.931</td>
</tr>
<tr>
<td>s.e.($d$)</td>
<td>0.093</td>
<td>0.092</td>
<td>0.091</td>
<td>0.093</td>
</tr>
<tr>
<td>GSP</td>
<td>0.894*</td>
<td>0.904*</td>
<td>0.908*</td>
<td>0.927</td>
</tr>
<tr>
<td>s.e.($d$)</td>
<td>0.063</td>
<td>0.062</td>
<td>0.062</td>
<td>0.064</td>
</tr>
<tr>
<td>PMGPH</td>
<td>0.859*</td>
<td>0.867*</td>
<td>0.855*</td>
<td>0.856</td>
</tr>
<tr>
<td>s.e.($d$)</td>
<td>0.081*</td>
<td>0.080*</td>
<td>0.080*</td>
<td>0.082</td>
</tr>
</tbody>
</table>

Notes: GPH, GSP and PMGPH estimates are the Geweke and Porter-Hudak (1983) log periodogram, the Robinson (1995b) Gaussian semiparametric, and the Phillips’ variant Geweke and Porter-Hudak estimates, respectively; s.e. is the standard error of the estimate. For the GPH, GSP and PMGPH estimates, the known theoretical variance of innovations $(\pi^2/6)$, $(1/4m)$ and $(\pi^2/24)$ is imposed in the calculation of $d$, respectively. *, ** indicates significance at the 5 and 10 % levels, respectively. The hypothesis $H_0: d = 0$ is tested against the two-sided alternative of $d \neq 0$.

nonstationary. But, they exhibit long memory and mean-reverting behavior, implying that the PPP hypothesis is valid for Turkey according to the results of these estimators.

In addition to the fractionally integrated results, an alternative explanation for the mean reversion of each real exchange rate dynamics becomes useful. Thus, the persistence of each of the real exchange rate dynamics can be analyzed by using the impulse responses explained in Section 3. The impulse response coefficients at different time horizons after a unit shock can be computed for each real exchange rate series analyzed in the paper using an estimated ARFIMA model for each real exchange rate reported in Table 4. The estimated impulse response coefficients are reported in Table 6. Figures 1, 2, 3 and 4 display the plots of the impulse responses for the selected model for each real exchange rate. The plots of impulse responses for real TL/USD, real TL/DM, real TL/BP and real TL/FF show the same striking dynamics. As shown in Table 4, estimated $d$ values for these real exchange rate series are about 0.8, implying that these are covariance nonstationary and have long memory; for this reason, the mean reversion towards their equilibrium takes a much longer decay in these impulse response plots. The plot of the estimated impulse response coefficients shows that a unit shock has no permanent effect on the value of series. For this reason, these series will return to their equilibrium after an innovation. These findings are also consistent with Sarno (2000)’s and Erlat (2003)’s evidence in this respect.
Figure 1
Impulse Responses of Real TL/USD rate

Figure 2
Impulse Responses of Real TL/DM rate
Figure 3
Impulse Responses of Real TL/BP rate

Figure 4
Impulse Responses of Real TL/FF rate
Table 6
Impulse Responses

<table>
<thead>
<tr>
<th>Steps</th>
<th>TL/USD</th>
<th>TL/DM</th>
<th>TL/BP</th>
<th>TL/FF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1.106000</td>
<td>1.111000</td>
<td>1.105000</td>
<td>1.112000</td>
</tr>
<tr>
<td>3</td>
<td>1.074466</td>
<td>1.080685</td>
<td>1.072597</td>
<td>1.082297</td>
</tr>
<tr>
<td>4</td>
<td>1.031774</td>
<td>1.037647</td>
<td>1.029246</td>
<td>1.039618</td>
</tr>
<tr>
<td>5</td>
<td>0.996408</td>
<td>1.001613</td>
<td>0.993396</td>
<td>1.003812</td>
</tr>
<tr>
<td>6</td>
<td>0.968486</td>
<td>0.973070</td>
<td>0.965113</td>
<td>0.975431</td>
</tr>
<tr>
<td>7</td>
<td>0.946007</td>
<td>0.950082</td>
<td>0.942357</td>
<td>0.952563</td>
</tr>
<tr>
<td>8</td>
<td>0.927378</td>
<td>0.931038</td>
<td>0.923507</td>
<td>0.933615</td>
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<tr>
<td>9</td>
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<td>0.914858</td>
<td>0.907487</td>
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<tr>
<td>11</td>
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</tr>
<tr>
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<td>0.874855</td>
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<td>0.870403</td>
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<td>13</td>
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<td>0.867462</td>
<td>0.860534</td>
<td>0.870333</td>
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<td>0.851552</td>
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<tr>
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<td>0.850090</td>
<td>0.843317</td>
<td>0.853035</td>
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<td>19</td>
<td>0.820951</td>
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<td>0.811189</td>
<td>0.804752</td>
<td>0.814289</td>
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<td>22</td>
<td>0.804750</td>
<td>0.806004</td>
<td>0.799612</td>
<td>0.809124</td>
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<td>0.799921</td>
<td>0.801091</td>
<td>0.794740</td>
<td>0.804229</td>
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<td>0.790112</td>
<td>0.799579</td>
</tr>
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<td>25</td>
<td>0.790966</td>
<td>0.791980</td>
<td>0.785706</td>
<td>0.795152</td>
</tr>
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<td>0.787740</td>
<td>0.781501</td>
<td>0.790927</td>
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<td>0.773635</td>
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<td>0.766400</td>
<td>0.775752</td>
</tr>
</tbody>
</table>

5. Conclusion

This paper has examined the presence of mean reversion in the real bilateral Turkish lira exchange rate obtained between Turkey and its major trading partners. In this article, a variety of parametric and semiparametric methods for estimating the long memory parameter of the real bilateral Turkish exchange rate have been presented. The results of R/S and modified R/S statistics on real exchange rate series show that these series have long memory. The findings obtained from other semiparametric methods used in this article imply that the real exchange rate series are fractionally integrated and have a long memory, which
means that the series are mean-reverting processes. In sum, the findings of the semiparametric estimation on real exchange rate series suggest that real exchange rate series have long memory and are mean reverting processes.

In addition to the above findings, the real exchange rate has been examined by means of ARFIMA models and impulse response analysis. For this particular aim, initially, we estimated different models for each real exchange rate series by using the reduced form of the Whittle approximate maximum likelihood estimator. We have chosen SIC tests as a model selection criterion based on diagnostic tests on the residuals. As a result, an ARFIMA(1, 0.870, 0) is selected for TL/USD; an ARFIMA (1, 0.868, 0) for TL/DM, an ARFIMA(1, 0.868, 0) for TL/BP and an ARFIMA(1, 0.869, 0) for TL/FF. Evidence from these results show that all of them follow nonstationary processes; but, that they have long memory. The series are mean-reverting processes so that the effect of the shocks disappears in the long run. Finally, the findings of semiparametric and parametric estimates suggest that the real exchange rate series are mean reverting processes, implying that the PPP hypothesis is valid for Turkey. These results show consistency with those of Sarno (2000) and Erlat (2003).

References


Özet

Reel döviz kurunda ortalamaya geri dönme: Türkiye’den ampirik bulgular, 1980-1999