IS A CORRECTION NECESSARY FOR BETA ESTIMATION?

BETA TAHMİNİ İÇİN DÜZELTME GEREKLİ Mİ?

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ABSTRACT

The CAPM beta ($\beta$) is a parameter, which plays a central role in modern finance as a measure of an asset's systematic risk. Practitioners rely on beta estimates when estimating costs of capital, applying various valuation models, determining portfolio strategies and implementing risk management techniques. Researchers also rely on beta estimates for many applications such as determining relative risk, testing asset pricing models, testing trading strategies and conducting event studies.

As evidenced, betas are not stable over time. Hence, the instability of betas over time leads to important practical problems. If nonstationarity of $\beta$s is ignored, investors may make significant mistakes resulting in underestimating or overestimating systematic risk.

In this study, we have proposed two correction methods for beta estimation. Findings of our study suggest that proposed correction methods seem to provide accurate beta estimates.

Keywords: Systematic Risk, $\beta$ estimation, and Correction Methods.

ÖZET

Sermaye Varlıklarını Fiyatlama Modeli’ndeki (SVFM) $\beta$ katsayısı finans alanında sistematik risk ölçütü olarak önemli bir yere sahiptir. Piyasa aktörleri sermaye maliyeti hesaplamada, varlık değerlərmələrində, portföy stratejiləri belirləmedə ve risk yönetimində $\beta$ katsayısından yararlanırlar. Benzer şekilde, araştırmacılar da nispi risk belirləme, modellərin test edilməsi gəibi konularda $\beta$ katsayını kənardırələr.

Yapılan çalışma mərhələlərda ortaya konduğu üzere, $\beta$ katsayları zaman içinde istəklərən deyilir. Dolayısıyla, $\beta$ katsaylarının zaman içindeki değişikliyi uygulamada önemli problemlərarətəbilir. $\beta$ katsaylarının değişikliyi göz ardı etmərə xüsusiyyətlər yaranır kərələrənə alınan olabilir.

Bu çalışmada, $\beta$ katsayısı tahmini üçün düzeltmə yöntemlərini önerilmişdir. Çalışma bulğuları önerilen yöntemlərin daha doğru tahmin sonuçlarını verdiği göstərməktedir.

Anahtar Sözcüklər: Sistematik risk, $\beta$ tahmini, düzeltmə yöntemlər.
INTRODUCTION

In investment decisions, an important element frequently used is risk. Although dictionary defines risk as “hazard, peril, exposure to loss or injury”, with respect to investment, risk is considered in different terms. In the development of portfolio theory, Markowitz (1952) defined risk in terms of a well-known statistical measure known as the variance. Specifically, Markowitz quantified risk as the variance about an asset’s expected return.

Although the total risk of an asset can be measured by its variance, this risk measure can be divided into two general types of risk: systematic risk and unsystematic risk. William Sharpe defined systematic risk as the portion of an asset’s variability that can be attributed to a common factor. Sharpe defined the portion of an asset’s variability that can be diversified away as unsystematic risk (Fabozzi, 1999; 68).

The CAPM beta ($\beta$) is a parameter, which plays a central role in modern finance as a measure of an asset’s risk. Beta coefficient known as systematic risk measure compares the variability of an asset’s historical returns to the market as a whole. That is, beta measures an asset’s expected change for every percentage change in the benchmark index (Clarfeld and Bernstein, 1997). As Karacabey (2001) and Maximiliano (2001) point out, while making investment decisions, investors are concerned only with the systematic risk, which is the risk of the market as a whole, because the unique risk (unsystematic risk) is diversified away by a well-balanced portfolio. For this reason, $\beta$ is the only concern investors have when they value securities.

Although the capital asset pricing model (CAPM) developed by Sharpe (1964) and Lintner (1965) assumes that the beta coefficient is constant through time, as early studies have shown; betas are not stable over time. Beta estimation is still an important issue in emerging markets like Turkey due to the fast changing nature of the financial markets. Therefore, in this study, we propose two correction methods for beta estimation. The objective of this study is to see if the proposed methods provide accurate results in the Istanbul Stock Exchange (ISE). The study first reviews the literature on beta estimation, followed by procedures of data collection and methods. The paper concludes with discussions on the study results, and implications.

LITERATURE REVIEW

Much effort has been devoted to beta estimation. Some examples of studies on beta estimation include the stability of beta over time (Harvey, 1989; Cheng and Boasson, 2004), time horizons of investors (Levhari and

Some research focused on the stability of beta estimates across time. Blume (1971) and Levy (1971) reported low correlations for betas through time. Blume (1975) studied whether estimated betas exhibit a tendency to regress towards the great mean of all betas. Cheng and Boasson (2004) used a time weighted least square method to estimate betas of emerging markets and found that the betas for these markets do shift over time. However, beta instability and potential asymmetry are examined by Braun et al. (1995) and found weaker evidence of time-varying betas.

Such studies as Baesel (1974), Altman et al. (1974), Blume (1975), and Roenfeldt (1978) showed that the longer the estimation periods, the more stable the beta estimates become.

Fabozzi and Francis (1978), Sunder (1980), Alexander and Benson (1982), Lee and Chen (1982), Ohlson and Rosenberg (1982), Bos and Newbold (1984), and Collins et al. (1987) suggested that the beta of securities is not stable but is best described by some type of stochastic parameter model.

ESTIMATING BETA

The estimation of systematic risk (or ‘beta’) is critical to many applications in finance. Practitioners rely on beta estimates when estimating costs of capital, applying various valuation models, determining portfolio strategies and implementing risk management techniques. Researchers also rely on beta estimates for many applications such as determining relative risk, testing asset pricing models, testing trading strategies and conducting event studies.

Since beta coefficient is unobservable, a great deal of energy has been devoted to its estimation. The unobservability of $\beta$ can be resolved by simply regressing an asset’s return on the return to the market portfolio using time-series data as long as the asset returns are stationary. However, Groenewold and Fraser (1999) states that asset returns may not be stationary in practice, resulting in $\beta$ instability over time. Earlier studies on beta estimation found that portfolio betas are not stable and tend to regress toward 1 over time (Blume, 1971, 1975). The economic logic here is that underlying riskiness of a firm tends to move toward the riskiness of the average firm.

The instability of $\beta$s over time leads to important practical problems. Apart from those posed by the interpretation of $\beta$s, which change over time,
there are problems of estimation both for practical use and for use in testing the CAPM. Estrada (2000) points out that if nonstationarity of returns is ignored, investors may make significant mistakes resulting in underestimating or overestimating systematic risk (and overestimating or underestimating risk-adjusted returns).

There are at least two sources of beta instability (Fabozzi, 1999; 75). The first is statistical estimation error, having to do with such things as the length of the time interval over which returns are measured (e.g., daily, monthly, or quarterly). Nothing in the theory indicates whether weekly, monthly, or even daily returns should be used. Nor does theory indicate any specific number of observations, except that statistical methodology requires that more observations will give a more reliable measure of beta.

Another source of apparent beta instability has to do with the use of beta as a single index of systematic risk. As Lee and Jang (2007) and Avramov and Chordia (2006) indicate, securities have multiple sources of systematic risk. Therefore, any single risk measure that attempts to aggregate all sources of systematic risk can appear to be unstable when it encompasses one or more of the macroeconomic or microeconomic sources of systematic risk that are changing.

A common approach to estimating beta is to apply the standard market model estimated under OLS (Ordinary Least Square). However, as Cohen et al (1983), Frankfurter (1994), and Brailsford and Josev (1997) point out, many different beta values can be obtained for the one stock, depending on the frequency of return data and length of the analysis period.

One of the ways used in investigating beta stability is to estimate the betas over sub-periods and compare the estimated betas for these shorter samples with each other and with the full sample betas. Groenewold and Fraser (1999) indicate that results of prior studies found out that there was considerable variability of betas over time.

Beta is generally estimated by using the standard market model, which is expressed as the following:

\[ R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it} \]  

where \( R_{it} \) is the realized return on security \( i \) over return interval \( t \); \( R_{mt} \) is the realized return on the market index over return interval \( t \); \( \alpha_i \) is the constant term for security \( i \); \( \beta_i \) is the sensitivity of security \( i \) returns to the market index returns measured as \( \text{cov}(R_i, R_m) / \text{var}(R_m) \). \( \beta \) is usually measured over a finite number of return measurement intervals using OLS and is assumed to be constant. \( \varepsilon_{it} \) is the error/residual term for security \( i \) for return interval \( t \), \( \varepsilon_{it} \sim N(0, \sigma_{it}^2), \text{cov}(\varepsilon_{it}, \varepsilon_{it-1})=0, \text{cov}(\varepsilon_{it}, R_{mt})= 0. \) \( t \) is return interval over which the return is measured \( t= 1,2,3,\ldots\ldots.,T. \) The return interval can be
expressed as any period of time. It is common for $t$ to be measured on a
daily, weekly or monthly basis.

As Beer (1997) indicates, since the simple OLS produces biased beta
estimates, there needs to be correction on the beta estimates. Several
methods have been proposed for estimating beta coefficients in these
circumstances. Among these methods, the models proposed by Blume
(1971), Vasicek (1973), Scholes and Williams (1977), Dimson (1979), and
Cohen et al. (1980) are the best known. Blume (1971) suggests a correction
method, which requires regressing the estimated values of $\beta$ in one period
on the values estimated in a previous period and using this estimated
relationship to modify betas for the future evaluations. Scholes and
Williams (1977) showed that a consistent beta estimator can be obtained by
a model based on the lag beta, lead beta, the current or synchronous beta,
and the first order serial correlation coefficient for the index. Dimson
(1979) established that the systematic risk estimate can be obtained by
aggregating the coefficients of a multiple regression.

Cohen et al. (1983) proposed the use of a three-pass regression to
estimate the asymptotic value that the OLS beta approaches as the
differencing interval is lengthened. Vasicek (1973) suggests correcting beta
estimates using Bayesian method.

Beer (1997) pointed out that several studies tested the effectiveness of
these adjustment techniques and they concluded that each technique
produces beta estimates that slightly reduce the amount of bias.

**DATA AND METHODOLOGY**

The main purpose of this study is to suggest correction methods for
beta estimates. The sample includes 90 stocks listed on the Istanbul Stock
the study, we used monthly return data of 90 stocks. The data were taken
from database of ISE.

We proposed two three-stage methods for beta correction. The first
method requires estimating betas based on the relationship between two
periods such as $t$ and $t+1$. Note that period $t+1$ is inclusive of period $t$.
Consider, $t$ represents a 5-year period between years 1994-1999, then $t+1$
turns out to be a 6-year period which covers the years 1994-2000. In the
study, we conducted the analysis for $t=5, 6, 7, 8, 9$ provided that 1994 is
the starting year. In the first stage, standard estimates of betas for different
time periods (i.e. 5-year, 6-year …and 10-year) are obtained by using the
standard market model given in Equation (1). In the second stage, a
regression was run based on cross-sectional data obtained in the first stage by using the Equation (2).

\[ \beta_i^{t+1} = a + b \beta_i^t \]  

(2)

where \( \beta_i^{t+1} \) is the beta for security i over return interval \( t+1 \) and \( \beta_i^t \) is the beta for security i over return interval \( t \). In the third stage, betas are estimated for individual stocks using above relationship (Equation 2). Then, estimated betas were compared to the observed betas.

The second method requires estimating betas based on the relationship between two n-year periods such as \( t_1 \) and \( t_2 \) where \( n=5, 6, 7, 8, 9 \). Note that the last \( n-1 \) years of \( t_1 \) and the first \( n-1 \) years of \( t_2 \) coincide. Consider a situation where \( n=5 \), \( t_1 \) represents a 5-year period between years 1994-1999, and then \( t_2 \) turns out to be a 5-year period which covers the years 1995-2000 where the latter includes the last four years of the former. In the first stage, standard estimates of betas for different time periods (i.e. 5-year, 6-year ...and 10-year) are obtained by using the standard market model given in Equation (1). In the second stage, a regression was run based on cross-sectional data obtained in the first stage by using the Equation (3).

\[ \beta_i^{t_2} = a + b \beta_i^{t_1} \]  

(3)

where \( \beta_i^{t_2} \) is the beta for security i over return interval \( t_2 \) and \( \beta_i^{t_1} \) is the beta for security i over return interval \( t_1 \). In the third stage, betas are estimated for individual stocks using above relationship (Equation 3). Then, estimated betas were compared to the observed betas.

EMPIRICAL RESULTS

Table 1 presents the regression results for Method 1. Here, we obtained statistically significant results and high R²s. Regression results indicate that coefficients converge to 1 as the time period is lengthened. This finding implies that for accurate estimates of beta, it is necessary to run regression between longer time periods such as 8-9 Years.
Is a Correction Necessary for Beta Estimation?

Table 1: Regression Results for Method 1

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Equation</th>
<th>Beta Estimates</th>
<th>Adjusted-R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-6 Year</td>
<td>$\beta_{94-00} = 0.157 + 0.830\beta_{94-99}$</td>
<td>(3,305) (16,490)</td>
<td>0.753</td>
</tr>
<tr>
<td></td>
<td>$\beta_{95-01} = 0.106 + 0.874\beta_{95-00}$</td>
<td>(5,313) (39,960)</td>
<td>0.946</td>
</tr>
<tr>
<td></td>
<td>$\beta_{96-02} = 0.049 + 0.960\beta_{96-01}$</td>
<td>(2,750) (47,324)</td>
<td>0.962</td>
</tr>
<tr>
<td></td>
<td>$\beta_{97-03} = 0.024 + 0.975\beta_{97-02}$</td>
<td>(1,718) (63,596)</td>
<td>0.978</td>
</tr>
<tr>
<td>6-7 Year</td>
<td>$\beta_{94-01} = 0.108 + 0.872\beta_{94-00}$</td>
<td>(5,184) (39,359)</td>
<td>0.946</td>
</tr>
<tr>
<td></td>
<td>$\beta_{95-02} = 0.043 + 0.966\beta_{95-01}$</td>
<td>(2,458) (49,809)</td>
<td>0.965</td>
</tr>
<tr>
<td></td>
<td>$\beta_{96-03} = 0.020 + 0.980\beta_{96-02}$</td>
<td>(1,711) (74,981)</td>
<td>0.984</td>
</tr>
<tr>
<td>7-8 Year</td>
<td>$\beta_{94-02} = 0.027 + 0.982\beta_{94-01}$</td>
<td>(1,551) (52,452)</td>
<td>0.969</td>
</tr>
<tr>
<td></td>
<td>$\beta_{95-03} = 0.016 + 0.983\beta_{95-02}$</td>
<td>(1,439) (78,379)</td>
<td>0.986</td>
</tr>
<tr>
<td>8-9 Year</td>
<td>$\beta_{94-03} = 0.018 + 0.980\beta_{94-02}$</td>
<td>(1,536) (78,897)</td>
<td>0.986</td>
</tr>
</tbody>
</table>

t-test values in parentheses

We have compared the estimated betas with observed betas for the time period considered in order to test the effectiveness of the Method 1. For this purpose, first, we estimated betas for individual stocks using obtained regression models. Then, we calculated the sum of squared errors obtained by taking differences between estimated betas and observed betas. As Figure 1 presents, sum of squared errors tend to decrease as time period is lengthened.

![Figure 1: Comparison of Estimated and Observed Betas (Method 1)](image)
Table 2 presents the regression results for Method 2. For this method, we again obtained statistically significant results and high $R^2$s. Regression results indicate that coefficients converge to 1 as the time period is lengthened even though some fluctuations exist in the value of coefficients. This finding implies that for accurate estimates of beta, it is necessary to run regression between longer time periods such as 8-9 Years.

Table 2: Regression Results for Method 2

<table>
<thead>
<tr>
<th></th>
<th>$\beta_{t2}=a+b\beta_{t1}$</th>
<th>Adjusted-$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-5 Year</td>
<td>$\beta_{95-00} = 0.143 + 0.796\beta_{94-99}$</td>
<td>0.612, n=5</td>
</tr>
<tr>
<td></td>
<td>$\beta_{96-01} = 0.083 + 0.884\beta_{95-00}$</td>
<td>0.931</td>
</tr>
<tr>
<td></td>
<td>$\beta_{97-02} = 0.109 + 0.901\beta_{96-01}$</td>
<td>0.832</td>
</tr>
<tr>
<td></td>
<td>$\beta_{98-03} = 0.036 + 0.978\beta_{97-02}$</td>
<td>0.970</td>
</tr>
<tr>
<td>6-6 Year</td>
<td>$\beta_{95-01} = 0.063 + 0.880\beta_{94-00}$</td>
<td>0.848, n=6</td>
</tr>
<tr>
<td></td>
<td>$\beta_{96-02} = 0.022 + 0.974\beta_{95-01}$</td>
<td>0.954</td>
</tr>
<tr>
<td></td>
<td>$\beta_{97-03} = 0.058 + 0.946\beta_{96-02}$</td>
<td>0.906</td>
</tr>
<tr>
<td>7-7 Year</td>
<td>$\beta_{95-02} = -0.035 + 1.012\beta_{94-01}$</td>
<td>0.932, n=7</td>
</tr>
<tr>
<td></td>
<td>$\beta_{96-03} = -0.001 + 0.989\beta_{95-02}$</td>
<td>0.976</td>
</tr>
<tr>
<td>8-8 Year</td>
<td>$\beta_{95-03} = -0.033 + 1.000\beta_{94-02}$</td>
<td>0.924, n=8</td>
</tr>
</tbody>
</table>

$t$-test values in parentheses

We have compared the estimated betas with observed betas for the time period considered in order to test the effectiveness of the Method 2. For this purpose, first, we estimated betas for individual stocks using obtained regression models. Then, we calculated the sum of squared errors obtained by taking differences between estimated betas and observed betas. As Figure 2 presents, sum of squared errors tend to decrease as time period is lengthened.
Is a Correction Necessary for Beta Estimation?

When the two methods in our study are compared, Method 1 seems to be a more accurate correction method in beta estimations because sum of squared errors between observed and estimated betas in Method 1 is less than that of Method 2 (Figure 3).

CONCLUSION

The beta is a parameter, which plays a central role in modern finance as a measure of an asset's systematic risk. Since practitioners, researchers also rely on beta estimates for variety of purposes, the estimation of systematic risk (or 'beta') is really critical in finance.

As evidenced, betas are not stable over time. Hence, the instability of betas over time leads to important practical problems. Apart from those posed by the interpretation of $\beta$s that change over time, there are problems of estimation both for practical use and for use in testing the CAPM. If
nonstationarity of returns is ignored, investors may make significant mistakes resulting in underestimating or overestimating systematic risk.

As the instability of betas over time is the case, there needs to be correction on the beta estimates. In this study, we proposed two methods for beta correction. Proposed methods seem to provide statistically significant results for accurate beta estimates. In practice, generally 5-year period is considered for beta calculations based on monthly data. However, as our study shows, longer time periods such as 8-9 Years give better beta estimations.

REFERENCES


